

APPENDIX MOPTICAL POWER SPECTRUM ANALYZER

The power spectrum analyzer is a device intended to indicate the amplitude and direction of the frequency components of the image structure of a transparency. If the transparency is a negative or positive photograph, the analyzer will immediately give the resolving power of the photographic system used (the maximum frequency obtained by the analyzer), and the relative content and distribution of the frequency components passed by the photographic system. Different types of scenes will have different distributions, although any given distribution will have a whole class of scenes corresponding to it.

THEORY OF OPERATION:

The operation of the power spectrum analyzer is based on the fact that the Fraunhofer image of a point source produced by a diffracting screen is the Fourier transform of the wave amplitude in the plane of the screen. If  $f(x, y)$  represents the wave amplitude in the plane of the diffracting screen and  $\psi(k_x, k_y)$  represents the amplitude of the image where  $(k_x, k_y)$  is the normalized coordinate of a point in the image plane, then

$$\psi(k_x, k_y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{i(k_x x + k_y y)} dx dy \quad \text{Eq. (1)}$$

In practice  $f(x, y)$  is limited to the pupil of an optical system and is zero outside the pupil, and the integral can be restricted to the area of the pupil. It can be seen that  $k_x$  and  $k_y$  which are position coordinates in the image plane correspond to frequency components in the plane of the diffracting screen.

The value of the normalized coordinate is given by

$$k = \frac{2\pi}{\lambda} \sin \theta \quad \text{Eq. (2)}$$

where  $\lambda$  is the wavelength of the light and  $\theta$  is the angle of diffraction. The actual position  $(x', y')$  in the image plane as a function of the actual spatial frequency component  $(\nu_x, \nu_y)$  in the plane of the diffracting screen is given by

$$x' = f \tan \theta_x, \quad y' = f \tan \theta_y, \quad \text{and} \quad \text{Eq. (3a)}$$

$$\sin \theta = \lambda \nu. \quad \text{Eq. (3b)}$$

The wave amplitude  $f(x, y)$  may be complex; that is, it may represent both variations in the magnitude and phase of the wave across the pupil containing the diffracting screen. This phase variation can have a strong effect on the image and is unwanted in the power spectrum analyzer. The manner of its control and the degree to which it can be tolerated will be discussed later. For the present we shall assume that the phase variation is negligible.

Although the phase variation may be negligible in the pupil, phase variations will exist in the image plane depending on the particular geometry of the diffracting screen.

It should be emphasized that the function we are analyzing is the amplitude of the wave in the pupil as modified by the diffracting screen. The transmission of a screen is usually in terms of the intensity of the light transmitted which is the square of the amplitude transmitted if there is no phase variation.

The diffracted image observed is also the intensity of the wave in the image plane rather than the amplitude, and is the product of the amplitude and its complex conjugate.

The amplitude in the image plane is the Fourier transform of the amplitude in the pupil plane. Thus the intensity of the image is the product of this Fourier transform and its complex conjugate. This product is commonly known as

the power spectrum (thus the name of the instrument) and is the Fourier transform of the autocorrelation function of the diffracting screen.

Since the power spectrum has eliminated the phase information (the product of a function and its complex conjugate is the square of the modulus of the function), any given power spectrum corresponds to a whole class of objects which differ from each other only in the manner in which the structural details are distributed, and thus is a statistical measure of the structural content of the object.

The power spectrum analyzer is intended to measure the structural content of the image of a transparency (as measured by its amplitude transmission) by presenting its power spectrum graphically as the variation in intensity of a point image resulting from diffraction by the transparency. The intensity at any point in the image plane is directly proportional to the power spectrum density of that frequency component of the transparency having a frequency proportional to the distance of the point from the center of the image and a direction determined by the direction of the point from the center of the image.

#### DESCRIPTION OF INSTRUMENT:

Figure M-5 illustrates diagrammatically the layout of the instrument. A monochromatic light source, pinhole, and collimator serve to provide monochromatic plane waves incident on the transparency. An immersion tank is provided to reduce phase variations in transmission to a minimum. A collector lens forms an image of the pinhole after the light has been diffracted by the transparency.

This image is the power spectrum desired, and a sheet of film inserted here would record the image. However, in most cases the center of the image, corresponding to zero frequency, contains an appreciable fraction, frequently more

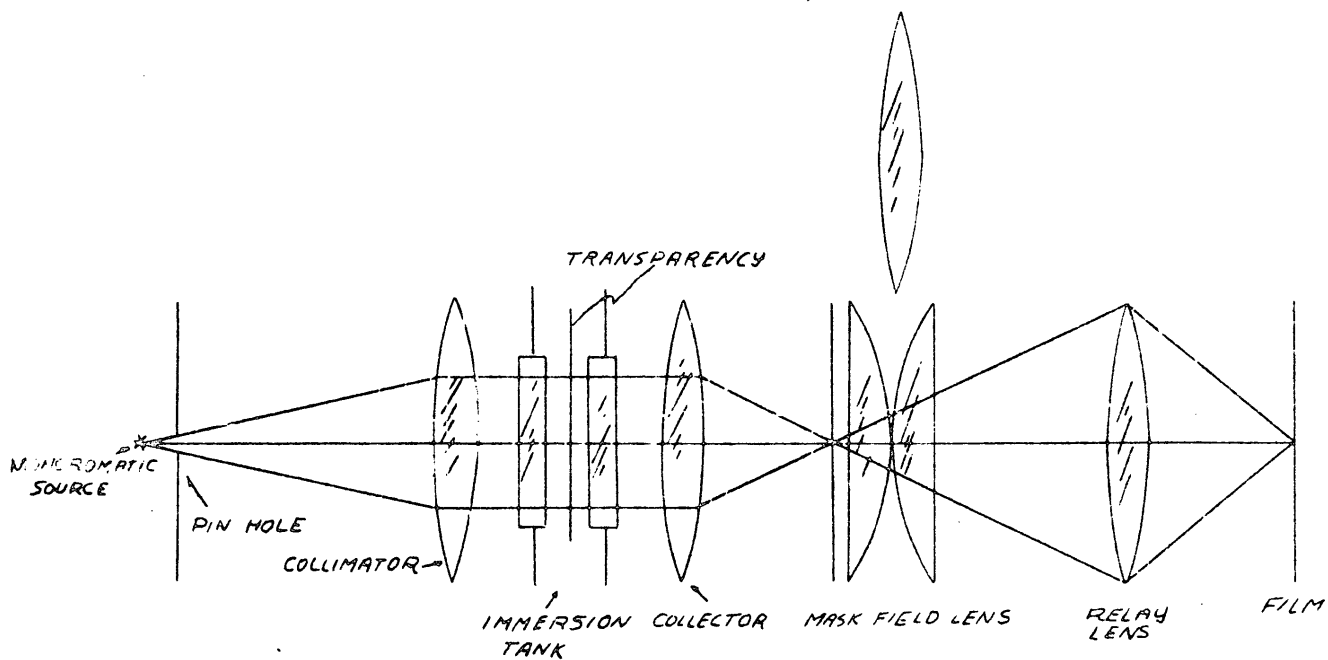


FIG M-5

than half, of the total energy, and the rest is distributed over a large area. If the exposure is sufficient to record the major part of the power spectrum, the center will be heavily overexposed and much of the power spectrum will be spread over by irradiation from the center image.

To reduce this effect to a minimum a mask consisting of an opaque central spot is inserted in this image plane and the image is relayed to a more convenient location by a relay lens.

The calibration of the frequency scale is most easily done by inserting a grating of known frequency, such as a Ronchi grating, in the immersion tank and recording its image. The fundamental and its harmonics will be recorded, providing an accurate frequency reference.

A check on the reliability of the system can be made by taking a picture of the image when the transparency is removed. If the mask is also removed, the point image obtained will represent the weighting function of the power spectrum measurement. The intensity at any point in the image plane is not strictly proportional to the power spectrum density of the corresponding frequency, but to a weighted average for that frequency and its neighboring frequencies. The spread of the point image with the transparency removed will represent the weighting function.

The main effect of enlargement of this image will be a corresponding change in the weighting function. This includes slow variations in phase, such as occurs in the presence of aberrations. Such slow variations will have no high frequency components. However, another effect of the slow phase variations is a weighting of the areas of the transparency in their contributions to the power spectrum. If the transparency does not have homogeneous texture, different aberration distributions will result in slightly different power spectra.

If the power spectrum itself is not expected to have fine structure, a relatively large weighting function is tolerable. The most efficient use of the system is

tolerance is the enlargement of the pinhole. The pinhole image is compact so that the weighting does not extend too far out, but the increase in transmitted energy, and consequent reduction in exposure time is considerable.

A picture taken with the transparency removed but the mask in place will show how much light will be spread by the strong zero-frequency component in spite of the central mask. This residual light is a result of the unlimited, although low-level, diffraction spread; scattered light, and out-of-focus ghost images. The diffraction spread is generally very small, the scattered light can be kept at a minimum by having a clean and well-baffled optical system, and the ghost images can be kept to a minimum by means of low-reflection coatings.

#### EXPERIMENTAL RESULTS:

Figures M-1, M-2, M-3 and M-4 show some power spectra obtained with this instrument along with contact prints of the corresponding transparencies which were used as diffracting screens, and enlargements of selected portions of the transparencies.

The power spectrum of the Ronchi grating shows the fundamental and harmonics of the structure. If the lines and spaces were exactly equal in width, the even harmonics would be absent. Their presence, although they are noticeably weak, indicates that the lines and spaces are not exactly equal. This grating was used to calibrate the frequency scale.

Two of the remaining pictures were taken with a relatively low-resolution system and the third with a high-resolution system. The low-resolution system was limited to about 13 lines per mm. The high resolution system was limited to about 90 lines per mm.

One of the low-resolution pictures is of a river bed with foliage and the other is of a developed area. The river-bed picture has a power spectrum consisting of a broad, generally undifferentiated spread with a diffuse bar across

it in one direction. The broad spread indicates a generally random structure. The bar correlates with the direction of illumination of the scene, the shadow being very roughly linear and oriented in the same direction.

The power spectrum of the developed area shows strong spikes oriented along the principal directions of the artificial structure with almost no power associated with the frequencies oriented otherwise. One of those principal spikes shows nodes indicating regular periodic structure covering an appreciable area of the transparency. On examination it can be seen that plowed fields correspond to the nodes.

The high-resolution picture is also of a developed area, and this is again reflected in the strongly spiked structure of the power spectrum. This picture also has a plowed field, but the relative area is small and the nodes are not apparent.